Topological Quantum Field Theories and Topological Recursion

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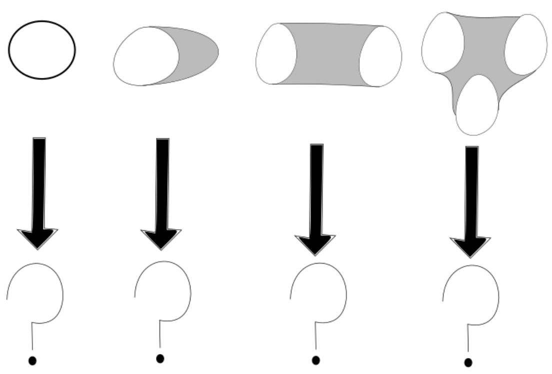
Topological Quantum Field Theory (TQFT)

A (d+1)-dimensional Topological Quantum Field Theory is an association of (d+1)-dimensional (smooth compact orientable) manifolds with boundary to linear functionals. There are various axioms that make this association invariant under diffeomorphisms.



Graphical Method of Calculation

We can represent surfaces by trivalent graphs. Take a pair of pants decomposition and let each pair of pants be a vertex then let the boundary components (glued together or not) be represented by edges. We can use this to help determine the linear functionals of the theory.

Topological Recursion

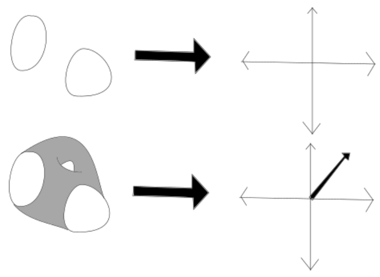
One can define symplectic invariants on a plane curve C given some immersion (x,y) (like an embedding but with possible self intersection). They are denoted $F^g = \omega_0^g$. They follow the following recursion

$$\omega_1^0(z_1) = y(z_1)dx(z_1) \qquad \omega_2^0(z_1, z_2) = \frac{dz_1 dz_2}{(z_1 - z_2)^2}$$

$$\omega_{n+1}^g(z_1, z_S) = \Sigma_{\mathbf{a}} Res_{\mathbf{a}} K(z_1, z)[\omega_{n+2}^{g-1}(z, \overline{z}, z_S) + \Sigma_{I \sqcup J = S}^{g_1 + g_2 = g} \omega_{|I|+1}^{g_1}(z, z_I)\omega_{|J|+1}^{g_2}(\overline{z}, z_J)]$$

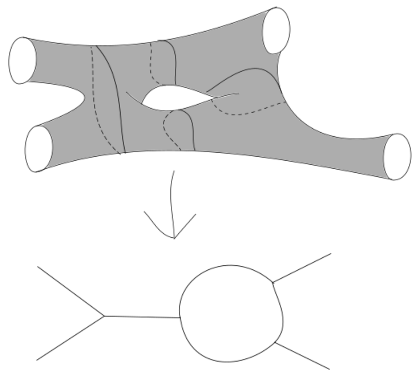
To the right is a diagrammatic interpretation of this recursion. This can be associated a graphical representation as well.

[1] R.J. Lawrence, An Introduction to Topological Quantum Field Theory
[2] M. Atihay, The geometry and physics of knots



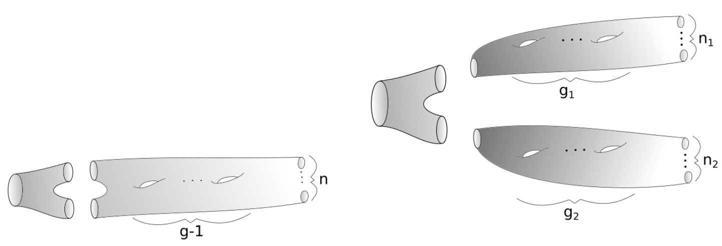
Classification of 2-dimensional TQFT

There is a complete classification of surfaces with boundary. They are determined by the genus, g, and the number of boundary components, n. We also know that a boundary component is diffeomorphic to a circle. We can also build every surface out of a pair of pants (three holed sphere). Therefore the left determines our TQFT.



Research

The aim of our research was to see and give a elementary explanation to a Topological Quantum Field Theory that is contained inside the invariants described on the left. The approach we were using was through the graphical methods of both the Topological Quantum Field Theories and the Topological Recursion.



$$\omega_{n+2}^{g-1}(z, \overline{z}, z_S) \qquad \omega_{n_1+1}^{g_1}(z, z_I)\omega_{n_2+1}^{g_2}(\overline{z}, z_J)$$

References

[3] P. Norbury, String and dilaton equations for counting lattice points in the moduli space of curves
[4] B. Eynard, N. Orantin, Invarients of algebraic curves and topological expansion.