Exploring the significance of betweenness centrality in granular networks.

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Trick question: What would be a commonality between jars of sand, organizational relations, ecological food webs, a city's power grid, a series of roads and the internet?

The answer is that they all can be modelled with great fidelity as networks and analysed to a great extent using graph theoretical properties. That is to say, that we can look at these systems as a collection of actors (nodes) and their relations with other actors in the system (links or edges).

For granular systems, this means we are now analysing the system in physical space and state space (a contact network or graph detailing the individual particle's kinematics) and determining its evolution over time as it is subjected to axial loading.

Dr. Tordesillas' past research in this field details the detection of dominant shear bands (where failure occurs) prior to nucleation (genesis) using graph theoretic properties. This has been extremely successful in showing that the signature of the dominant shear band is present prior to peak stress, which goes against the grain of traditional soil mechanics [1].

My research was focussed on a graph theoretic property called betweenness centrality, which is calculated as follows:

$$[2] \qquad C_b(i) = \sum_{i \neq j \neq k} \frac{g_{jk}(i)}{g_{jk}}$$

i.e the ratio of geodesics (shortest paths) from any nodes j to k passing through node i and the total number of geodesics from j to k.

Betweenness centrality has been part of the standard arsenal of centrality measures for networks and can reveal some interesting features about a system. Sociologically speaking, being in a position of high betweenness is equivocal to being a mediator of exchanges between people/organisations in a system (e.g an employment agency usually mediates exchanges between employers and the agency's workers).

One of the key assumptions used to measure betweenness, is the notion that information or flow, will opt to travel along the shortest paths in a system. This evidently makes betweenness inaccurate for certain social networks, where information can travel through random interactions. In the context of granular systems however, this is a key assumption. If betweenness measurements succeed in revealing the shear band prior to nucleation, then this will aid the hypothesis that forces travel along shortest paths.

During this program, I used data detailing the individual grain kinematics and contacts to construct a contact network with weighted edges. This was then used to compute betweenness centrality for all 5098 grains over 299 strain intervals. These results were then plotted and compared with other properties such as the minimum cut to provide an overall picture of how effective betweenness is with regards to detection of shear bands.

Results have showed some promise; as noted by the plots on the right. The plots (red/coral indicates high betweenness (top10%), purple indicates semi-high (between top 10%-20%) and green indicates low betweenness) show that the diagonal (dominant) shear band is already being tracked by betweenness very early in the loading stage. This persistence of the dominant shear band is a strong indicator of the power of network analysis to compliment and inform soil mechanical methods.

For me, this concept of using techniques from fields such as traffic modelling and neuro-science in granular mechanics really highlights the true interdisciplinary spirit of applied mathematics. It is hoped that these results will push granular problems into the field of optimal control where we can manipulate the shear band. This will be truly ground-breaking!

Scholarship Experience:

Undertaking this scholarship opportunity is something I cannot recommend enough for an undergraduate student interested in mathematics. More than just a nifty summer job, it gives students an opportunity to learn and engage in learning that is liberated from assessment and exams. Unlike coursework, research generally doesn't have a pre-made solution for reference and instead, constant care and revision must be made in order to truly bring out the fruits of your pursuit.

This experience also taught me the true value of independent learning and collaboration. Throughout my research, I was required to constantly teach myself about various topics, which included programming and computational skills. I was made to constantly check my methods and interact with other post-docs and PhD students as well as engage in critical discussion through meetings and presentations. Thanks to the program, I have gained valuable skills in research that will definitely aid me in future academic endeavours.

References:

[1] Tordesillas A, Walker DM, Andò E, Viggiani G. 2013 Revisiting localized deformation in sand with complex systems. Proc R Soc A 469: 20120606. http://dx.doi.org/10.1098/rspa.2012.0606

[2] Gabor Csardi, 'Package 'igraph'' 2014-04-22 23:08:29 http://cran.r-project.org/web/packages/igraph/igraph.pdf

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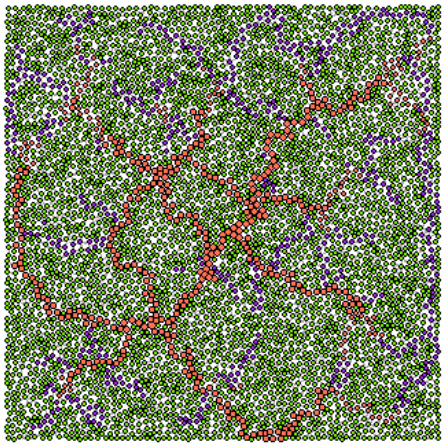

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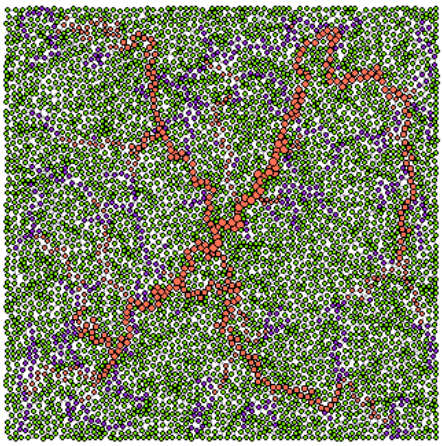

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